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LETTER TO THE EDITOR

Comment on "There Is No Error in the Kleiser–Schumann Influence Matrix Method"

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The letter "There Is No Error in the Kleiser–Schumann Influence Matrix Method" by Kleiser, Härtel, and Wintergerste [1] makes two important points: (1) the original method presented by Kleiser and Schumann in 1980 [2] is indeed correct [3]; and (2) my *alternate* formulation of the tau correction is similar in spirit but different in detail from Kleiser and Schumann's original method. These conclusions immediately follow from the single sentence in [1]: "The latter approach was adopted by Kleiser and Schumann, who applied the correction algorithm to each of the three partial solutions of the B-Problem." Unfortunately this statement (or equivalent wording) does not appear in any of the earlier literature on tau corrections [2, 4, 5], and its absence led to my development of the alternate formulation.

A researcher who decides he or she would like to implement the tau correction to the influence-matrix method therefore now knows (at least) two alternate implementations exist. Which method should such a researcher employ? Without hesitation my recommendation would be for the formulation presented in [1]; here is why.

By correcting partial solutions separately, linear combinations of these solutions are guaranteed to be divergence free. This fact makes the method described in [1] much more flexible than the one I proposed in [3] when different boundary conditions are handled.

To illustrate, consider the approach when a stress-free top and a no-slip bottom is desired. First, note that even and odd Chebyshev modes are coupled for this problem. Therefore, employing the formulation I presented in [3] involves a tau-correction solution which has even and odd modes coupled to each other. Such an implementation is unnecessarily cumbersome and requires a Chebyshev solver for the Helmholtz equation that allows even and odd modes to be coupled.

In contrast, when implementing the algorithm presented in [1], mixed boundary conditions can be handled trivially by summing three partial problems, each of which involves only Dirichlet conditions and each of which, therefore, has even and odd Chebyshev modes decoupled. The tau correction for each of these partial problems will be identical and will

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also have even modes decoupled from odd. Therefore, with this method coupling only becomes apparent when the influence matrix is solved, which is absolutely trivial.

Despite my suggestion that readers should employ the tau correction of [1] instead of that presented in [3], I would encourage anyone interested in implementing the tau correction to read all of the references cited below, especially [1–3]. Although some details involving the tau correction differ, many other details are identical, and much can be learned by digesting all of information now available.

I would like to acknowledge Dr. Keith Julien who implemented the mixed-boundary formulation of the tau correction that I describe here. Future publications will report the solutions we have computed with this technique.

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